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LITERATURE CITED

1. L. S. Leibenzon. Manual of Oil-Field Mechanics [in Russian], Part 1, GNTI, Moscow (1931), p. 25.
2. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Vol. 2, Fizmatgiz, Moscow (1963).
3. M. P. Volarovich and A. M. Gutkin, "Flow of viscoplastic material between two parallel plane walls and in annular space between two coaxial tubes," Zh. Tekh. Fiz., 16, No. 3 (1946).
4. M. P. Volarovich and A. M. Gutkin, "Flow of viscoplastic disperse systems in gap between two coaxial tubes," Kolloidn. Zh., No. 25 (1963).
5. A. Kh. Mirzadzhanzade, Questions of Hydrodynamics of Viscoplastic and Viscous Liquids in Application to Petroleum Extraction [in Russian], Azorneftneshr, Baku (1959).
6. A. A. Movsumov, R. S. Gurbanov, N. A. Gasan-Zade, and R. K. Iskenderov, "Investigation of motion of a clay solution in an annular gap," Izv. Vyssh. Uchebn. Zaved., Neft' Gaz, No. 4, 29-31 (1974).
7. M. A. Sattarov, "Some models of filtration in porous media," Dokl. Akad. Nauk SSSR, 203, No. 1 (1972).
8. M. A. Sattarov, "A study of the special features of flow through porous media," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, 5 (1973).
9. M. A. Sattarov, "Hydrodynamic method of classification of liquids in porous media," Dokl. Akad. Nauk TadzhSSR, 19, No. 11 (1976).
10. S. F. Aver'yanov, "Water permeability of soil and ground in relation to their air content," Dokl. Akad. Nauk SSSR, 69, No. 2 (1949).

CONCENTRIC IMPACT OF POINTED BODIES

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In [2] the concentric press described in [1] was analyzed for the limiting case of a sphere composed of a set of narrow pyramids which occupy the sphere not continuously but with a certain porosity $K > 1$ (K is the ratio of the volume of the sphere to the total volume of the pyramids). Whereas [2] was concerned with the static action of the press, the present article deals with the dynamic process of compression in which the pyramids approach each other at a certain speed. This question arose as a natural extension of the work described in [2]. As before, the entire effect is self-similar, the compression of the material at the center of the device is infinitely great and lasts a finite time (until externally relieved). For the parts in the center not to be destroyed, it is sufficient to assume slight linear hardening of the press material under pressure; experiments [3] show that under pressure the strength increases considerably.

Diagrams showing the device at the initial moment and at a later stage are presented in Fig. 1a, b. The pyramids approach the center at the rate u_0 . In the center there is formed a spherical zone of continuous compression whose boundary moves outwards at the rate v ; behind it a shock wave spreads out from the center at velocity w . We note that the porosity $K = (\beta/\alpha)^2$, where α is the angle at the vertex of the uncompressed pyramid, and β is the angle at the vertex of the compressed pyramid.

Figure 2 shows the path of a lateral particle of the pyramid up to the closing of the gap. Clearly, $-u_0\alpha = v(\beta - \alpha)$ ($u_0 < 0$), whence $u_0/v = -(\sqrt{K} - 1)$, which for low porosity ($K - 1 = \epsilon \ll 1$) gives $u_0/v = -\epsilon/2$.

A qualitative picture of the motion is given in Fig. 3. Until the pyramids close up, the material moves at a constant rate (from q to r_0); this is followed by a smooth deceleration along the path from r_0 to r_1 . At the shock wave the velocity decreases abruptly but re-

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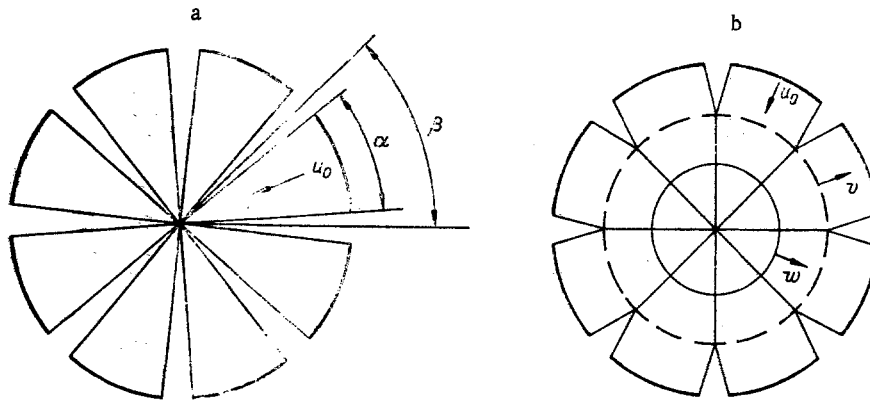


Fig. 1

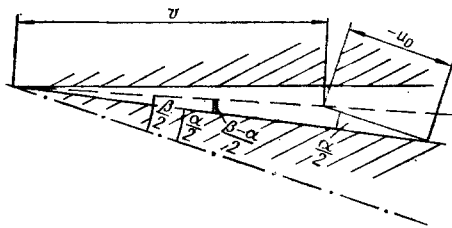


Fig. 2

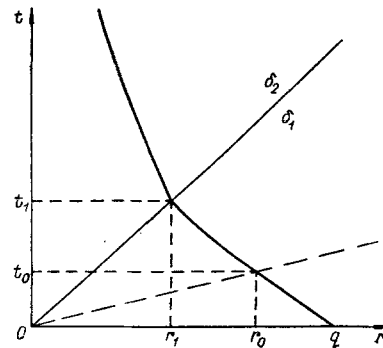


Fig. 3

mains directed towards the center, and the point approaches the center asymptotically. The compression of the material ahead of the shock wave is equal to δ_1 , behind the shock wave it is equal to δ_2 , and in the center it is always infinite,

We take the density dependence of the pressure for isotropic compression in the usual form

$$p = \frac{\rho_0 c_0^2}{3} (\delta_i^3 - 1), \quad (1)$$

where ρ_0 and c_0 are the initial density and speed of sound; $\delta_i = \rho_i / \rho_0$. The Young's modulus $E \sim \rho_i c^2$, but with dependence (1) $\rho_i \sim c$, i.e., $E \sim \rho_i^3$ or

$$E = E_0 \delta_i^3. \quad (2)$$

The region of elasticity of the material in the relative strains e_x, e_y, e_z is shown in Fig. 4. The material does not fail in isotropic compression, i.e., on the diagonal of the cube and in a narrow elongated neighboring zone.

In what follows we will represent the compression of the material as a combination of isotropic compression (not necessarily slight) and a small elastic deformation. For simplicity we set the Poisson's ratio $\mu = 1/3$; in this case $E_0 = \rho_0 c_0^2$.

In Fig. 5 we have shown the initial position and a subsequent position of an element of a pyramid (q, dq, α) and (r, dr, β). The stresses along (p) and across (σ) the radius are different. Under the pressure p the element is isotropically compressed by a factor δ_i and, at the same time, extended along the arc by the stress $p - \sigma$. For its new dimensions we obtain the relations

$$dr = \frac{dq}{\delta_i^{1/3}} \frac{1}{1 + \frac{2p - \sigma}{3E}}; \quad (3)$$

$$\beta r = \frac{\alpha q}{\delta_i^{1/3}} \left(1 + \frac{2}{3} \frac{p - \sigma}{E} \right). \quad (4)$$

Since α and β can be arbitrarily small, all the velocities are directed along radii, i.e., the motion in the continuous zone is spherically symmetric and one-dimensional.

The mass conservation equation takes the form

$$K\delta = \partial q^3 / \partial r^3, \quad (5)$$

where δ is the true compression, which is related to the isotropic compression δ_i by the condition

$$\frac{\delta_i}{\delta} = 1 + \frac{2}{3} \frac{p - \sigma}{E}. \quad (6)$$

Euler's equation takes the form

$$\frac{du}{dt} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{2}{\rho r} (p - \sigma) = 0. \quad (7)$$

We will seek the solution of system of equations (3)-(7) in self-similar form; as the variable we will take $\lambda = q/c_0 t$. We introduce the dimensionless radius $\varphi = r/q$ and velocity

$$\zeta = \frac{u}{c_0} = \frac{1}{c_0} \frac{\partial}{\partial t} (q\varphi) = -\lambda^2 \varphi'. \quad (8)$$

Using the fact that $\partial r / \partial q = \varphi + \lambda \varphi'$, from (5) we obtain

$$d\varphi/d\lambda = (1/\lambda)(1/K\delta\varphi^2 - \varphi). \quad (9)$$

In order to write Eq. (7) in the self-similar variables we use relations (1)-(4), (6), and (8) and obtain

$$\begin{aligned} \frac{du}{dt} &= -\frac{q}{t^2} \left[\frac{\lambda \delta'}{K \delta^2 \varphi^2} + \frac{2}{K \delta \varphi^3} \left(\frac{1}{K \delta \varphi^2} - \varphi \right) \right], \quad \frac{1}{\rho} \frac{\partial p}{\partial r} = \\ &= \frac{3c_0 K^{13/4} \varphi^{11/2} \delta^{7/2}}{2t} \left[\frac{\delta}{\lambda} \left(\frac{1}{K \delta \varphi^2} - \varphi \right) + \varphi \delta' \right], \\ \frac{2}{\rho r} (p - \sigma) &= \frac{3c_0^2 K^{9/4} \varphi^{7/2} \delta^{7/2}}{2q} (K^{3/4} \varphi^{3/2} \delta^{1/2} - 1). \end{aligned}$$

Finally, Eq. (7) is converted to the form

$$\frac{d\delta}{d\lambda} = \frac{\delta}{\lambda} \frac{\left(\frac{1}{K \delta \varphi^3} - 1 \right) \left(\frac{2\lambda^2}{K \delta \varphi^3} - \frac{3}{2} K^{13/4} \delta^{9/2} \varphi^{11/2} \right) - 3K^{9/4} \delta^{7/2} \varphi^{5/2} (K^{3/4} \delta^{1/2} \varphi^{3/2} - 1)}{\frac{3}{2} K^{13/4} \delta^{9/2} \varphi^{11/2} - \frac{\lambda^2}{K \delta \varphi^3}}. \quad (10)$$

Using (9), we obtain Eq. (8) for the velocity in the form

$$\zeta = -\lambda(1 - K\delta\varphi^3)/K\delta\varphi^2.$$

Thus the problem reduces to solving the two differential equations (9) and (10).

We start by investigating the asymptotic behavior in the center ($\lambda \rightarrow 0$). We find the solution for φ and δ in the form

$$\varphi = F\lambda^n, \quad \delta = D/\lambda^{3n}. \quad (11)$$

From (9) it follows that in this case

$$D(0) = 1/K(1 + n)F^3(0). \quad (12)$$

Substituting the expressions for φ and δ , Eqs. (11) and (12), in (10), as $\lambda \rightarrow 0$ we find that the solution exists and that the unknown exponent n must satisfy the relation

$$K^{1/4} = (1 + 2n)/\sqrt{1 + n},$$

which for low porosity ($K - 1 = \varepsilon \ll 1$) gives

$$n = \varepsilon/6.$$

Thus, at the center in the first approximation

$$\varphi \approx F(0)\lambda^{\varepsilon/6}, \quad \delta \approx D(0)/\lambda^{\varepsilon/2}. \quad (13)$$

As was to be expected, the exponent is the same as for the static press.

It is also possible to find the following approximation for the dependences $\varphi(\lambda)$ and $\delta(\lambda)$ near the center. Omitting the lengthy calculations, we present only the result for small n

$$\varphi = F(0)\lambda^n \left(1 + \frac{n}{15} F^8(0)\lambda^{2+8n} + \dots \right), \quad \delta = \frac{1 - \frac{n}{3} F^8(0)\lambda^{2+8n} + \dots}{F^3(0)K(1+n)\lambda^{3n}}.$$

The correction with respect to the asymptotic solution is quite small (of the order of λ^{2+8n}); i.e., the asymptotic relations hold over a broad region. The initial value $F(0)$ depends on the velocity ζ_0 ; the method of determining it is indicated below.

When $K = 1$ (continuous sphere), the problem can be simplified; in particular, it is clear from the equation that behind the shock wave there is a state of rest; i.e., φ and δ are constant.

We will determine the boundary conditions of the system (9), (10) on the contact boundary of the lateral faces of the pyramids $\lambda_0 = q/c_0 t$, $\varphi(\lambda_0) = \varphi_0$, $\delta(\lambda_0) = \delta_0$. It is clear from Fig. 2 that

$$\varphi_0 = \frac{v}{v - u_0} = \frac{1}{1 - \frac{u_0}{v}} = \frac{1}{1 - K}.$$

At the moment of contact the element is not compressed: $\delta_0 = 1$. Moreover

$$\lambda_0 = \frac{q}{c_0 t} = - \frac{q u_0}{c_0 (q - q \varphi_0)} = - \frac{\zeta_0}{1 - \varphi_0} = - \zeta_0 \frac{1 - \sqrt{K}}{1 - K - 1},$$

where $\zeta = u_0/c_0$.

Thus, the solution ahead of the shock wave is obtained by integrating Eqs. (9), (10) with the initial conditions

$$\lambda_0 = -\zeta_0 \sqrt{K}/(\sqrt{K} - 1), \quad \varphi_0 = 1/\sqrt{K}, \quad \delta_0 = 1.$$

At the center, however, only the asymptotic behavior of the solution is known

$$\varphi \approx F(0)\lambda^n, \quad \delta \approx D(0)/\lambda^{3n},$$

where $D(0)$ and $F(0)$ are related by expression (12) but the coefficients themselves are still unknown. They can be found using the relation at the shock wave. The construction of the solution is shown qualitatively in Fig. 6. At the point of intersection of the curves $\varphi(\lambda)$ from the outside and from the center (A_2) the following relation should be satisfied:

$$p_2 - p_1 = \rho_1(u_2 - u_1)(w - u_1).$$

the subscript "1" denotes the value ahead of the shock wave, the subscript "2" the value behind the shock wave. After transformation, with allowance for the fact that

$$w = \frac{r}{t} = \frac{r}{q} \frac{q}{t} = \varphi \lambda c_0, \quad p = \frac{\rho_0 c_0^2}{3} (\delta^3 - 1) = \frac{\rho_0 c_0^2}{3} (K^{9/4} \varphi^{9/2} \delta^{9/2} - 1),$$

$$u = c_0 \zeta = -c_0 \lambda \left(\frac{1}{K \delta \varphi^2} - \varphi \right),$$

we obtain

$$\frac{(K \varphi^2)^{17/4}}{3 \lambda^2} (\delta_2^{9/2} - \delta_1^{9/2}) = \frac{1}{\delta_1} - \frac{1}{\delta_2}. \quad (14)$$

The satisfaction of this condition is obtained by trial and error. The dependence $\varphi(\lambda)$ is constructed numerically inward from the contact front, and several $\varphi(\lambda) = F(\lambda)\lambda^n$ curves for various values of $F(0)$ are constructed from the center to meet it (see Fig. 6). The satis-

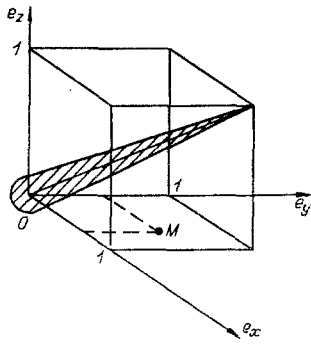


Fig. 4

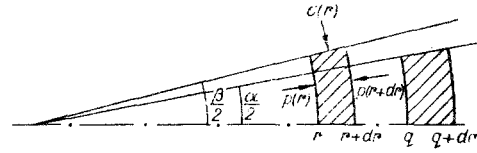


Fig. 5

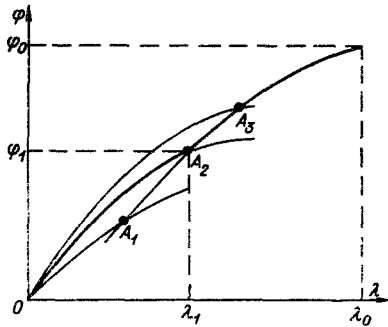


Fig. 6

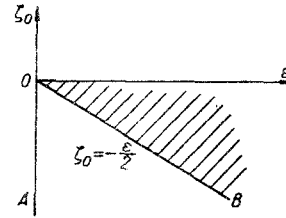


Fig. 7

fraction of (14) is tested for the points A_1 , A_2 , A_3 , and the point at which this condition is met is selected. In practice this is complicated, since the angles of intersection in Fig. 6 are very small; accordingly we will confine ourselves to considering the case of low velocities, for which everything can be solved analytically.

Before deriving the solution, we will first establish its region of applicability. To make sense physically ϵ must be greater than or equal to zero. Moreover, the shock wave proceeding from the center should not overtake the pyramid contact front; the minimum impact velocity $\zeta_0 = -\epsilon/2$ (see Fig. 2). The region of applicability AOB is indicated in Fig. 7. Using the smallness of ϵ and ζ_0 , we will calculate the compression of the material δ_2 at the shock wave in this region. We will determine δ_2 for the two extreme cases $\zeta_0 = -\epsilon/2$ and $\epsilon = 0$. The straight line OB corresponds to the case when the wave propagating from the center travels with the speed of sound c_0 : i.e., there is no compression and $\delta_2 = \delta_1 = 1$.

We will now solve the problem in the absence of porosity ($\epsilon = 0$). In this case we know the general solution for the velocity potential

$$\psi(r, t) = [f_1(r + c_0 t) + f_2(r - c_0 t)]/r,$$

and $u = \partial\psi/\partial r$. We will determine f_1 and f_2 for this case. At $t = 0$, $u = \text{const} = u_0$. In region 1 (Fig. 8) there is smooth deceleration, in region 2 a state of rest and $\delta \equiv \delta_2$ (there $\partial\psi/\partial r = 0$ or $\psi = \text{const}$). Without loss of generality, we set $\psi = 0$ i.e., $f_1(r + c_0 t) + f_2(r - c_0 t) = 0$, which is satisfied only when $f_1 = -f_2 = \text{const}$. Without loss of generality, we set $f_1 = f_2 = 0$. The value of f_1 depends on $r + c_0 t$; on crossing the wave 1-2 it does not change; i.e., $f_1 = 0$ in both regions. It remains to find f_2 and ψ in region 1. We have $\partial\psi(r, 0)/\partial r = u_0$, i.e., $\psi(r, 0) = u_0 r$, $f_2(r) = r\psi(r, 0) = u_0 r^2$, and consequently, $f_2(r - c_0 t) = u_0 (r - c_0 t)^2$ and $\psi(r, t) = u_0 (r - c_0 t)^2/r$. The velocity in region 1

$$u = \frac{\partial\psi}{\partial r} = u_0 \left[\frac{2(r - c_0 t)}{r} - \frac{(r - c_0 t)^2}{r^2} \right] = u_0 \left[1 - \left(\frac{c_0 t}{r} \right)^2 \right].$$

The displacement of the boundary up to the wave 1-2

$$S = \int_0^{Q_I/c_0} u dt = u_0 \int_0^{Q_I/c_0} \left[1 - \left(\frac{c_0 t}{r} \right)^2 \right] dt = \frac{2}{3} \frac{u_0}{c_0} Q_I.$$

The density behind the wave $\delta_2 = 1 - 3S/Q_I = 1 - 2\zeta_0$ ($\zeta_0 < 0$). Knowing δ_2 on the two boundaries

of the region AOB, for the entire region near the coordinate origin we obtain $\delta_2 = 1 - 2(\zeta_0 + \epsilon/2)$. Finally, we will calculate the pressure in the compression zone $p = (E_0/3)(\delta_1^3 - 1)$. We obtain the relation between the isotropic compression δ_1 and the true compression δ by multiplying (3) and (4) and substituting $\partial q/\partial r$ from (5)

$$\delta_1 = \delta^{3/2} \varphi^{3/2} K^{3/4}. \quad (15)$$

Taking into account the fact that (13) is a quite good approximation of $\varphi(\lambda)$ and $\delta(\lambda)$ and using (12), we obtain $\delta \varphi^3 = 1/[K(1+n)]$; substituting this in (15), we have $\delta_1 = (1 + \epsilon/6)\delta$.

The pressure at the center is at its maximum before the arrival of external unloading, i.e., at time $t = (Q_I/c_0)(1 - \epsilon/2\zeta_0)$ (Fig. 9); Q_I is the outside radius of the press. For this moment $\lambda_{\min} = q/[Q_I(1 - \epsilon/2\zeta_0)]$. Using this, we obtain an expression for the maximum pressure p_{\max} attainable at a point with given q

$$p_{\max} = \frac{E_0}{3} \left\{ \left(1 + \frac{\epsilon}{6}\right)^3 \left[1 - 2\left(\zeta_0 + \frac{\epsilon}{2}\right)\right]^3 \left[\frac{Q_I \left(1 - \frac{\epsilon}{2\zeta_0}\right)}{q} \right]^{\frac{3\epsilon}{2}} - 1 \right\}.$$

We select the porosity K , which is limited by the strength of the material, starting from the condition of preservation of the integrity of the material in the shock wave. From (3)-(5) it follows that

$$\frac{p - \sigma}{E} = \frac{3}{2} [K^{3/4} (\delta \varphi^3)^{1/2} - 1].$$

Substituting the approximate power-law expansion of $\delta(\lambda)$, $\varphi(\lambda)$ and the value $K^{1/4} = (1 + 2n)/\sqrt{1+n}$, we obtain

$$\frac{p - \sigma}{E} \approx \frac{3}{2} \frac{n}{1+n} \approx \frac{\epsilon}{4}. \quad (16)$$

The maximum shear stress $\tau = (p - \sigma)/2$ should not exceed the shear strength τ_* . For low compression, when $E \approx E_0$, the maximum porosity

$$\epsilon_{\max} = 8\tau_*/E_0 = 4p_*/E_0 \quad (17)$$

(p_* is the compressive strength).

It follows from (16) that in the entire region behind the shock wave the deformation of the material is the same (the shear angle $\gamma = \frac{3}{4}[(p - \sigma)/E] \approx \frac{3}{4}\epsilon$). It should be noted that as the center is approached the shear stress increases without bound, and the strength also increases strongly with pressure; accordingly the question of failure at the center of the press remains open. If the strength "overtakes" the increase in τ , then at $\epsilon < \epsilon_{\max}$ the material will be intact irrespective of the impact velocity ζ_0 . but at $\epsilon > \epsilon_{\max}$ it will fail at any arbitrarily small velocity. In the case postulated in [4] the porosity increases towards the center and exceeds ϵ_{\max} . With the method of compression described, such a device will fail, although in the already loaded state it might exist and withstand higher pressure than in our case. We also note that achieving dispersion of the pressure in the analogous cylindrical device mentioned in [4] is impossible, since near the axis of the cylinder the relative strains are large (point M in Fig. 4) and the material must fail.

Taking ϵ_{\max} in accordance with (17), we can calculate the pressure diagram. Figure 10 shows the results of the calculations for two values of the strength and the impact velocity (p_*/E_0 and ζ_0).

We will find the ratios of the dimensions of the impact and static devices Q_I/Q_S giving the same pressure at the same small radii q . For impact

$$\delta = [1 - 2(\zeta_0 + \epsilon/2)](2Q_I/q)^{\epsilon/2}, \quad (18)$$

and in the static case $\delta = \delta_*(Q_*/q)^{\epsilon/2}$, where Q_* is the radius of the continuous compression zone; $\delta_* = 1 + p_*/3E_0$ is the density at its boundary.

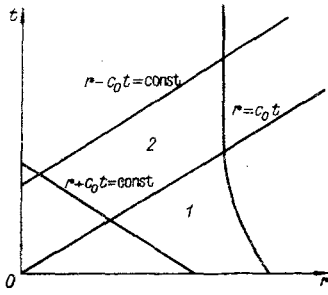


Fig. 8

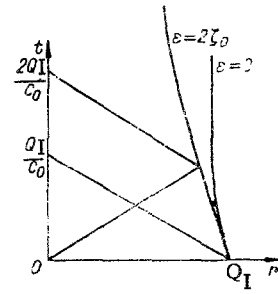


Fig. 9

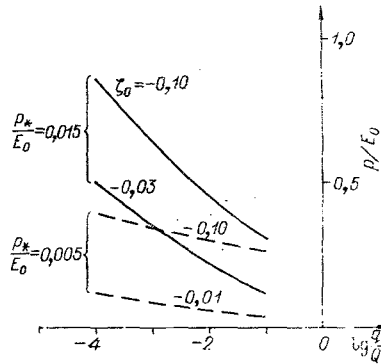


Fig. 10

For the most favorable porosity $\epsilon_{\max} = 4p_*/E_0$, $\delta_* = 1 + \epsilon/12$, and $Q_s/Q_s = \sqrt{p_0/p_*}$ (p_0 is the pressure at the surface of the press),

$$\delta = \left(1 + \frac{\epsilon}{12}\right) \left(\frac{Q_s}{q}\right)^{\frac{\epsilon}{2}} \left(\frac{p_0}{p_*}\right)^{\frac{\epsilon}{4}}. \quad (19)$$

Equating the δ from (18) and (19), we obtain

$$\left[1 - 2\left(\zeta_0 + \frac{\epsilon}{2}\right)\right] (2Q_I)^{\frac{\epsilon}{2}} = \left(1 + \frac{\epsilon}{12}\right) Q_s^{\frac{\epsilon}{2}} \left(\frac{p_0}{p_*}\right)^{\frac{\epsilon}{4}},$$

whence

$$\frac{Q_I}{Q_s} = \frac{\left(1 + \frac{\epsilon}{12}\right)^{\frac{2}{\epsilon}}}{2 \left[1 - 2\left(\zeta_0 + \frac{\epsilon}{2}\right)\right]^{\frac{2}{\epsilon}}} \sqrt{\frac{p_0}{p_*}}.$$

When $\epsilon \ll 1$ and $|\zeta_0| \ll 1$

$$\left(1 + \frac{\epsilon}{12}\right)^{\frac{2}{\epsilon}} \approx e^{\frac{1}{6}} \text{ and } \left[1 - 2\left(\zeta_0 + \frac{\epsilon}{2}\right)\right]^{\frac{2}{\epsilon}} \approx e^{-\left(\frac{4\zeta_0}{\epsilon} + 2\right)},$$

i.e.,

$$\frac{Q_I}{Q_s} = \frac{e^{\frac{4\zeta_0}{\epsilon} + \frac{13}{6}}}{2} \sqrt{\frac{p_0}{p_*}}.$$

In reality, $p_0/p_* < 0.5$, i.e.,

$$\frac{Q_I}{Q_S} < 0,35e^{\frac{4\zeta_0}{\epsilon} + \frac{13}{6}}$$

When $\zeta_0 = -\epsilon/2$ (extreme case for our press) this gives $Q_I/Q_S < 0.4$ and at twice the velocity ($\zeta_0 = -\epsilon$) $Q_I/Q_S < 0.06$.

Thus, even for a moderate approach velocity the central unit of the impact device is smaller than that of the static press.

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LITERATURE CITED

1. Kawai Naoto, "Production of very high pressure," J. Jpn. High Pressure Inst., 9, No. 3 (1971).
2. E. I. Zababakhin and I. E. Zababakhin, "A superhigh-pressure press," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1974)
3. L. F. Vereshchagin and V. A. Shapochkin, "Effect of hydrostatic pressure on the shear strength of solids," Fiz. Met. Metalloved., 9, No. 2 (1960).
4. Yu. I. Fadeenko, "A superhigh-pressure press," Zh. Prikl. Mekh. Tekh. Fiz.; No. 5 (1975).

MECHANISM FOR PLASTIC RELAXATION OF A SOLID IN A SHOCK WAVE

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§1. Model of Phenomenon

We consider plastic relaxation of a solid behind a stationary, plane shock front resulting from above-barrier slip of dislocations. Let the wave be moving in the direction of the x axis at a constant velocity D. We employ a coordinate system moving with the wave and we consider the state of an elementary plane layer of thickness dx which is stationary in this coordinate system. As is usual, we represent the actual dislocation ensemble by four effective slip systems of edge dislocations, the planes of which coincide with the planes of non-zero principal shear stresses (i.e., they make an angle of $\pi/4$ with the planes normal to the coordinate axes). We assume that in any elementary volume and for any slip system, an identical number of dislocations of opposite sign is created per unit time. However, the density of dislocations of opposite sign will not be the same in the elementary layer dx under consideration. Indeed, let the dislocation slip velocity be v. Then (from the assumed stationarity of the wave) the elementary layer dx crosses identical numbers of dislocations of opposite sign per unit time but it crosses them at different velocities: $(D + v/\sqrt{2})$ and $(D - v/\sqrt{2})$, respectively. Therefore, an excess of dislocations moving in the direction of the shock front will be observed in the layer. The relative magnitude of this excess is obviously $(v/D\sqrt{2})$. The effect of an excess of dislocations of one sign is equivalent to the presence in the layer dx of an equivalent Smith wall [1] which is the result of discontinuous relaxation because of a change in the principal strains ϵ_1 and ϵ_2 by an amount $(b\sqrt{2}/l)$, where b is the absolute value of the Burgers vector and l is the distance between dislocations belonging to a single set in the wall. The structure of a layer with a Smith wall is sketched in Fig. 1 with the dislocation density in the wall being exaggerated by several orders of magnitude for clarity. To understand what follows, it is important to emphasize that the Smith wall moves with a velocity D only in the formal sense; in fact, D is the displacement phase velocity of a section in which the dislocation density in the wall has a certain definite value whereas the excess dislocations themselves move with a velocity v. Besides the two sets of dislocations shown in Fig. 1, the wall contains yet another two sets of dislocations which are parallel to the plane of the figure so that the total number of dislocations per unit area of the wall is $4/l$. In the relief region behind the compression wave, the sign of v changes and the direction of the Burgers vector for the excess dislocations and for the Smith

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